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Dynamic Models

Inter-temporal optimization models are models that take into account multiple periods included in the planning horizon and are called dynamic models. In a dynamic framework, the stock of the resource in year t+1 is a function of both the decisions taken in year t and the autonomous progression of the resource from *t* to *t+1*. This relation of dependence is expressed as

$$\boldsymbol{x_{t+1} - x_t = g_t\,(x_t, u_t\,)}$$

For each period of time *t*, the system is described by a ***state variable*** **(x_t)** and a ***control variable*** **(u_t)**; the former represents the stock of the energy resource while the latter represents the extraction decision.

The standard intertemporal problem consists in determining the sequence of decisions (u_t) that maximize the net present value (NPV):

$$NPV = \sum_{t=1}^{T} \left(\frac{1}{1+r}\right)^{t-1} \pi_t(x_t, u_t) + \left(\frac{1}{1+r}\right)^{T} F(x_{T+1})$$

Subject to

$$x_{t+1} - x_t = g_t\,(x_t\,, u_t\,) \qquad\qquad t = 1,2,\ldots,T$$

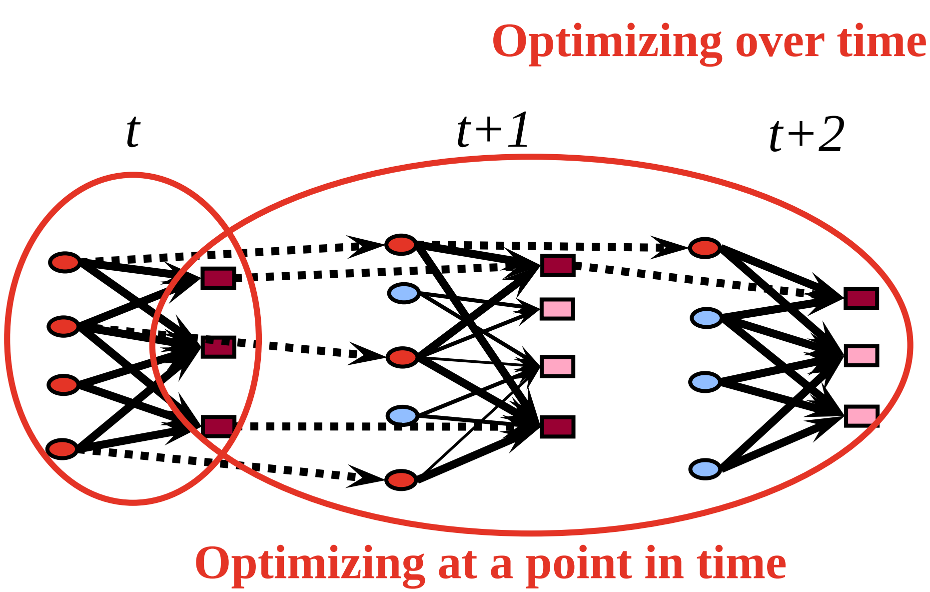
Dynamic Models

In addition to such an intertemporal model formulation that yields a single decision, there are **recursive sequential decision models where different decision stages are represented explicitly**. The essential difference with inter-temporal optimization models resides in their optimization method. Sequential decision models represent sequential decision-making with the gradual incorporation of information. **Rather than optimizing over the entire planning horizon, the optimization is performed for each stage individually**, though the results of stage t will influence the initial data in stage t+1.

The objective function for the recursive sequential model is formulated as:

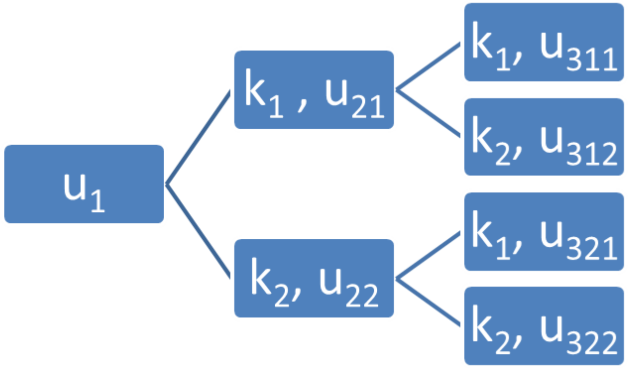
$$NPV_t = \max\left\{\pi_t(x_t, u_t) + \frac{1}{1+r} NPV_{t+1}(x_{t+1})\right\}$$

resembling dynamic programming



Stochastic Dynamic Models

A sequential stochastic decision problem can generally be represented by a decision tree. An example with three decision stages and two states of nature is presented below. Here, by starting from an initial state of the system decisions in stage 1 (u_1) are taken. Later, according to the state of nature occurring (k_1 or k_2), other decisions can be taken (u_{21} is, for example, the decisions taken in stage 2, taking into account the state of nature k_1).



Three decision stages and two states of nature

Stochastic Dynamic Models

In stochastic problems, one of the objective functions most frequently used is the mathematical expectation of total discounted profit:

$$NPV_t = \max\left\{E\big(\pi_t(x_t, u_t, k_t)\big) + \frac{1}{1+r} NPV_{t+1}(x_{t+1})\right\}$$

resembling **stochastic dynamic programming**.

Dynamic Stochastic Programming

In **dynamic stochastic programming**, on the other hand, a multi-stage approach is generally used implying that the agent takes several initial decisions (u_1) with uncertain knowledge of the future. This is followed by one of the states of nature (k) and the agent will take other decisions (u_{2k}) later on that depend on the decisions made in the first stage and the state of nature having occurred. In this case, the formulation becomes

$$\text{NPV}= \sum_{t=1}^{T} \left(\frac{1}{1+r}\right)^{t-1} \pi_t(x_t, u_t) + \left(\frac{1}{1+r}\right)^{T} \text{F}(x_{T+1})$$

from which the first-stage decisions (u_1) emerge. In the second stage,

$$\text{NPV}= \sum_{t=2}^{T} \left(\frac{1}{1+r}\right)^{t-1} \pi_t(x_{tk1}, u_t) + \left(\frac{1}{1+r}\right)^{T} \text{F}(x_{Tk1+1})$$

is used yielding second-stage decisions (u_2)

The Real Options Approach

Technology Adoption under Uncertainty: the value of waiting

Ex.: A solar panel that covers all your water heating requirements can be installed at \$1600. Your current water heating bill amounts to \$200, but next year it will change (your family will increase or decrease by 1 person with equal probability): with probability 0.5 it will rise to \$300, and with probability 0.5 it will fall to \$100. The bill will remain at this new level forever.

Is this a good investment ?

Assume an interest rate of 10%

The Real Options Approach

Ex.: A utility faces constant demand growth of 100 MW/year for two years. There are two alternatives for adding capacity:

1. Investing in a 200-MW coal-fired pp at a cap. cost of $180 million (Plant A)

2. Building a 100-MW oil-fired pp at a cap. cost of $100 million (Plant B)

Current O&M costs: $19 million/year for each 100 MW of Plant A
: $20 million/year for Plant B

The price of coal is fixed, but the price of oil will rise or fall next year with equal probabilities
➔Plant B O&M costs will become either 30 or $10 million per year

Half of plant A capacity can be sold 1 year after installation at a price of $90 million.

Which alternative to choose ? (interest rate = 10%)